

# Study on Free Flight Collision Risk in Free Flight Based on Conflict

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**Abstract:** Free flight is one of the effective methods to solve airspace congestion, so in order to ensure flight in free flight environment, the study of the collision risk is very important. Establish the collision risk model and give the adjustment quantity of course or velocity based on the conflict detection and liberation. The feasibility of collision risk model is verified through an example and the relationship between the risk of collision and the different separation is analyzed. So the pilot can choose the appropriate flight plan according to the adjustment quantity and the actual situation of the aircraft.

**Key words:** free flight; collision risk; course adjustment ;velocity adjustment

## I. Introduction

The concept of free flight was proposed in 1965<sup>[1-2]</sup> by William. The implementation of the free flight can not only save the flight time and fuel consumption, but also gain greater flight flow because of the full use of the airspace. At the same time, the possibility of collision increases with the increased flight flow. To ensure flight safety in free flight environment, the research on flight collision risk in free flight is particularly important.

Some fruit of free flight collision risk overseas has achieved. Some people like Rick Cassell, used the method of accident tree by quantitative relationship between aircraft collision risk and safe separation, and they gave the model of flight collision risk. Some scholars abroad also gave the model of airspace division sector using and aircraft meeting. The model can well finish the use of airspace sector, airspace evaluation and analysis of aircraft collision risk<sup>[3]</sup>. K.M. Feigh etc studied free flight deeply, and gave the conception of autonomous Mediterranean Free Flight (AMFF, as - Mediterranean Free Flight), and they studied the flying state in high density conditions especially, established and optimized the model of safety assessment based on AFMM<sup>[4]</sup>.

At home, the research of collision risk in free flight, mainly concentrated in the technology of airspace conflict detection and extrication. Zhang Zhao-ning, Cai Ming, Wang Li-li and Zhou Peng established the model of collision risk assessment in free flight and evaluated security of flight separation<sup>[5-9]</sup>. In 2012, Zhang Zhao-ning, Sun Chang established the model of collision risk based on the liberation of conflict and analyzed the relations between the conflict releasing Angle and collision risk<sup>[10]</sup>.

Based on the research of conflict detection and release, the model of collision risk based on the different solutions of conflict extrication, course adjustment and speed adjustment was established, using the principle of relative motion. At the same time, the adjustment was given on the different solutions. So the pilot can choose flight plan according to the actual situation.

## II. The establishment of coordinate system

The airspace of aircraft flying is three-dimensional space. So the separation is restricted to the transverse, longitudinal and lateral spacing. Using the coordinate of Descartes, the definition is as follows: the origin is one

point of the earth, the Z axis is vertical upward, the X axis is parallel to the horizontal and the Y axis is determined according to the right-hand rule. In the spatial domain, N aircrafts are in flight-  
 $A_1, A_2, \dots, A_i, \dots, A_j, \dots, A_n$ . Two aircrafts of them  $A_i, A_j$  are conflict detected and separation evaluated. At same time, the state of all aircrafts can be obtained. If existing positions of two aircrafts overlap in space position, there is the possibility of collision. Then the aircrafts need to be put out the conflict resolution to ensure flight safety.

To describe the problem for the convenience, two aircrafts are projected in XOY and XOZ plane respectively. The horizontal projective plane of aircrafts' protection zone is defined as that the center is the aircraft and the diameter of the area is S, as shown in figure 1. The vertical projective plane is the zone whose diameter is H, as shown in figure 2.

According to the principle of relative motion, the speed of aircraft  $A_j$  relative to  $A_i$  can be expressed in  $V_h$  for the horizontal plane and  $V_v$  for the vertical plane. If the protection zone of two aircrafts overlap, there is a collision. The pilot can adjust the course and speed to avoid collision. It can be safe only when the protection zone of aircraft  $A_j$  is in the front or rear of  $A_i$ .

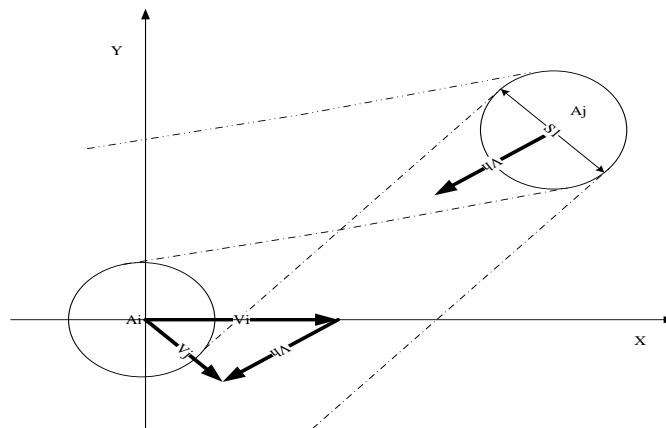


Figure 1 motion vector diagram of aircraft in XOY plane

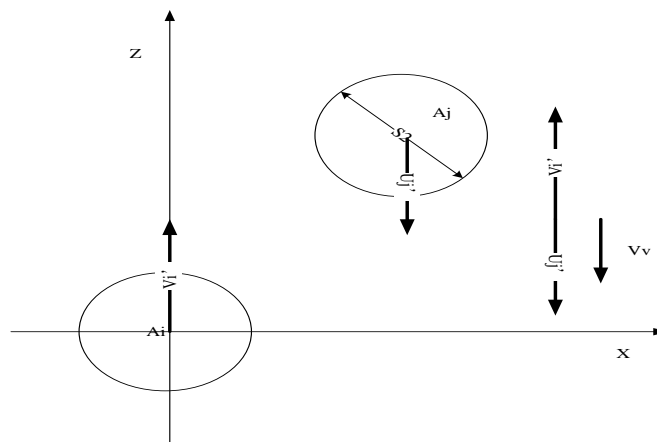


Figure 2 motion vector diagram of aircraft in XOZ plane

According to the principle of relative motion, the motion state is transformed to relative motion state: aircraft

$A_i$  do standard Brownian motion in original point and aircraft  $A_j$  is flying with relative speed  $V_h$  and  $V_v$ .

$\omega_1$  and  $\omega_2$  are positioning errors in a horizontal plane and vertical plane respectively, which obey normal distribution  $N_1(0, \sigma_1^2(t))$  and  $N_2(0, \sigma_2^2(t))$ .

### III. The establishment of collision risk model

#### 2.1 The establishment of collision risk model basing on course adjustment

The pilot can change course to avoid collision. Assuming in the flight process, only one aircraft in two aircrafts changes heading and the other one continues the original track flight. According to the common flight rules, if flight collision happens in the flight process, we only change the speed in horizontal plane instead of changing climbing and descending height of aircraft.

Figure 3 shows the course adjustment method to avoid conflict. The  $D$  is the initial distance between aircrafts in a horizontal plane and the  $h$  is the vertical distance in vertical plane.

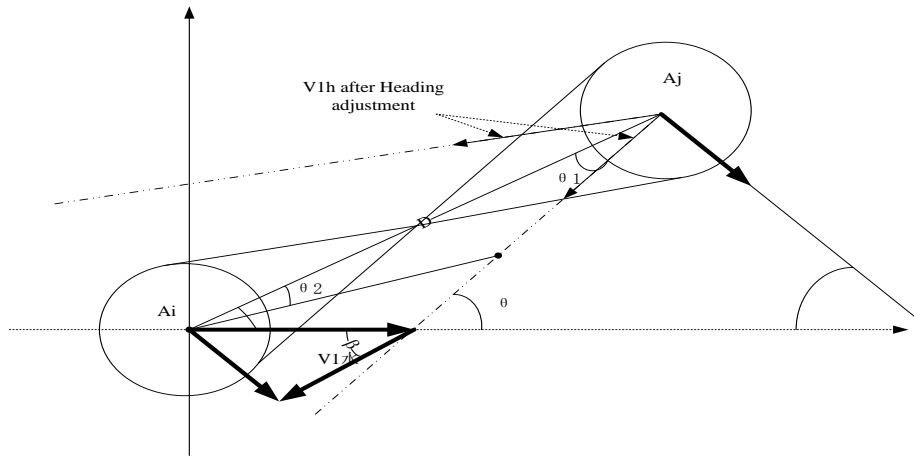


Figure 3 diagram of course adjustment method to avoid flight conflict

From figure 3, it is known that in the process of course adjustment, it need to keep the aircraft  $A_i$  course unchanged and only aircraft  $A_j$  course is adjusted. Define that the left-handed rotation is negative and the other is positive. So the adjustment of heading is  $\Delta\beta = \theta - \beta$ .

The distance in a horizontal plane between two aircrafts is as follows

$$X_1(t) = [D \cos \theta_1 - (\omega_h + V_{1h}t)] / \cos(\theta_2 + \theta_1)$$

The distance in a vertical plane between two aircrafts is as follows:

$$X_2(t) = h + V_{1v}t + \omega_v$$

$\omega$  obeys the standard normal distribution,  $N(0, \sigma^2(t))$ . The formula can be derived to:

$$X_1(t) = \left[ D \cos \theta_1 - (\omega_h + V_{1h}t) \right] / \cos(\theta_2 + \theta_1)$$

$$\square D \cos \theta_1 / \cos(\theta_2 + \theta_1) - V_{1h}t / \cos(\theta_2 + \theta_1) - N_1(0, \sigma_1^2) / \cos(\theta_2 + \theta_1) \\ = N_1((D \cos \theta_1 - V_{1h}t) / \cos(\theta_2 + \theta_1), \sigma_1^2 / \cos^2(\theta_2 + \theta_1))$$

$$X_2(t) = h + V_{1v}t + \omega_v \square h + V_{1v}t + N_2(0, \sigma_2^2) = N_2(h + V_{1v}t, \sigma_2^2)$$

Where  $u_h = (D \cos \theta_1 - V_{1h}t) / \cos(\theta_2 + \theta_1)$ ,  $\sigma_h^2 = \sigma_1^2 / \cos^2(\theta_2 + \theta_1)$ ,  $u_v = h + V_{1v}t$ ,  $\sigma_v^2 = \sigma_2^2$ .

The probability density function of distance for Aircraft in a horizontal and vertical plane can be derived as follows:

$$f_{X_1}(x) = \frac{1}{\sqrt{2\pi} \sigma_h(t)} e^{-\left[ \frac{(x-u_h)^2}{2\sigma_h^2(t)} \right]}$$

$$f_{X_2}(y) = \frac{1}{\sqrt{2\pi} \sigma_v(t)} e^{-\left[ \frac{(y-u_v)^2}{2\sigma_v^2(t)} \right]}$$

The probability of collision risk based on course adjustment in free flight is:

$$P_1(t) = P_{1h}(t) \bullet P_{1v}(t)$$

$P_{1h}(t)$  and  $P_{1v}(t)$  are the probability of collision risk based on course adjustment in a horizontal and vertical

plane. According to formula (5) and (6),  $P_{1h}(t)$  and  $P_{1v}(t)$  can be derived as:

$$P_{1h} = \int_{-S_1}^{S_1} f_{u_h}(x) dx = \frac{1}{\sqrt{2\pi} \sigma_h} \int_{-S_1}^{S_1} e^{-\frac{(x-u_h)^2}{2\sigma_h^2}} dx$$

$$P_{1v} = \int_{-S_2}^{S_2} f_{u_v}(y) dy = \frac{1}{\sqrt{2\pi} \sigma_v} \int_{-S_2}^{S_2} e^{-\frac{(y-u_v)^2}{2\sigma_v^2}} dy$$

$S_1$  and  $S_2$  are the minimum safety separations in the horizontal and vertical plane under free flight.

Every collision should be counted as two accidents. Considering in 107 flying hours, the probability CR of collision risk based on course adjustment can be expressed as:

$$CR = 2NP \cdot \int_0^{t_1} P_1(t) dt \square$$

NP is the number of aircrafts in each flight hour.

## 2.2 Establishment of collision risk model based on course adjustment

As shown in figure 4, when the protection zones of aircrafts overlap, the velocity of the aircrafts will be adjusted. For the convenience of building model, it is assumed that only the velocity of aircraft  $A_j$  is to be adjusted.

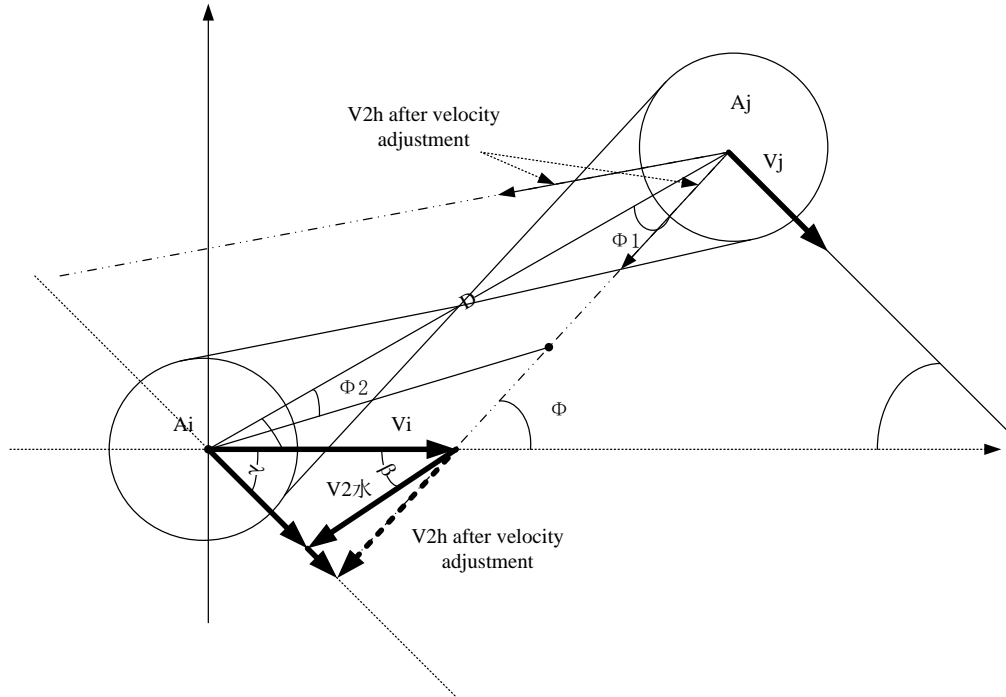


Figure 4 diagram of velocity adjustment method to avoid flight conflict

From figure 4, in the horizontal plane the relationship can be known:

$$\frac{V_i}{\sin(180 - \lambda - \beta)} = \frac{V_j}{\sin(\beta)}$$

$$\frac{V_i}{\sin(180 - \lambda - \phi)} = \frac{V_j'}{\sin(\phi)}$$

The formula of velocity adjustment is as follows:

$$\Delta V_j = V_j' - V_j = \frac{V_i}{\sin(180 - \lambda - \phi)} \cdot \sin(\phi) - \frac{V_i}{\sin(180 - \lambda - \beta)} \cdot \sin(\beta)$$

According to the collision avoidance mechanism, the distance formulas of velocity adjustment in the horizontal and vertical plane respectively is:

$$X_1'(t) = \left[ D \cos \phi_1 - (\omega_h + V_{2h}t) \right] / \cos(\phi_2 + \phi_1)$$

$$\square D \cos \theta_1 / \cos(\phi_2 + \phi_1) - V_{2h}t / \cos(\phi_2 + \phi_1) - N_1(0, \sigma_1^2) / \cos(\phi_2 + \phi_1)$$

$$= N_1((D \cos \phi_1 - V_{1h}t) / \cos(\phi_2 + \phi_1), \sigma_1^2 / \cos^2(\phi_2 + \phi_1))$$

$$X_2'(t) = h + V_{2v}t + \omega_v X_2(t) \square h + V_{2v}t + N_2(0, \sigma_2^2) = N_2(h + V_{2v}t, \sigma_2^2)$$

Where  $u_h' = (D \cos \phi_1 - V_{2h}t) / \cos(\phi_2 + \phi_1)$ ,  $\sigma_h^2 = \sigma_1^2 / \cos^2(\phi_2 + \phi_1)$ ,  $u_v' = h + V_{2v}t$ ,  $\sigma_{\perp}^2 = \sigma_2^2$ .

$P_2(t)$  is the collision probability based on velocity adjustment.  $P_{2h}(t)$  and  $P_{2v}(t)$  are the collision probability in horizontal and vertical plane respectively. So the collision probability can be expressed as:

$$P_2(t) = P_{2h}(t) \cdot P_{2v}(t)$$

The overlapping probability in horizontal and vertical plane are following respectively:

$$P_{2h} = \int_{-S_1}^{S_1} f_{u_h}(x) dx = \frac{1}{\sqrt{2\pi}\sigma_h} \int_{-S_1}^{S_1} e^{-\frac{(x-\mu_h)^2}{2\sigma_h^2}} dx$$

$$P_{2v} = \int_{-S_2}^{S_2} f_{u_v}(y) dy = \frac{1}{\sqrt{2\pi}\sigma_v} \int_{-S_2}^{S_2} e^{-\frac{(y-\mu_v)^2}{2\sigma_v^2}} dy$$

At the same time, the collision probability based on velocity adjustment is:

$$CR = 2NP \cdot \int_0^{t_2} P_2(t) dt$$

#### IV. Analysis example

##### 3.1 Determination of parameters

At present, the free flight has not yet been put into effect, so the relevant parameters can't be accurately put forward. Therefore, the example of this model draws on the experiences of the data of non-free flight. The relevant parameter is shown in table 1.

Assume that in free flight, aircraft is flying along a straight line in a period of time. The cruising speed of aircraft  $A_i$  and  $A_j$  is the same, on the cruising speed of Boeing 747-400. Horizontal speed is 485 knots and vertical speed is 0.05 knots. While the speed angle in the horizontal plane is sixty degrees, horizontal distance  $D$  is twenty nautical mile, and vertical distance is 0.15 nautical mile.

For the course adjustment, the amount of heading  $\Delta\beta$  is  $18^\circ$  and the collision risk is  $2.35 \times 10^{-9}$ . For the velocity adjustment, it is appropriate to change 160 knots in velocity and the collision risk is  $2.57 \times 10^{-9}$ . From results above, in this situation, course adjustment is in the range that the performance of the aircraft allows with the same probability collision. If the speed adjustment changes a lot, this model is not appropriate. Usually, the method of avoidance mechanism is to change course, the calculation results is also according with the actual situation.

##### 3.2 The influence of different separations to collision risk

Figure 5 shows the influence of different initial separations to flight collision risk based on the course adjustment. With the increase of the initial interval, collision risk reduces, which is consistent to the present air traffic situation.

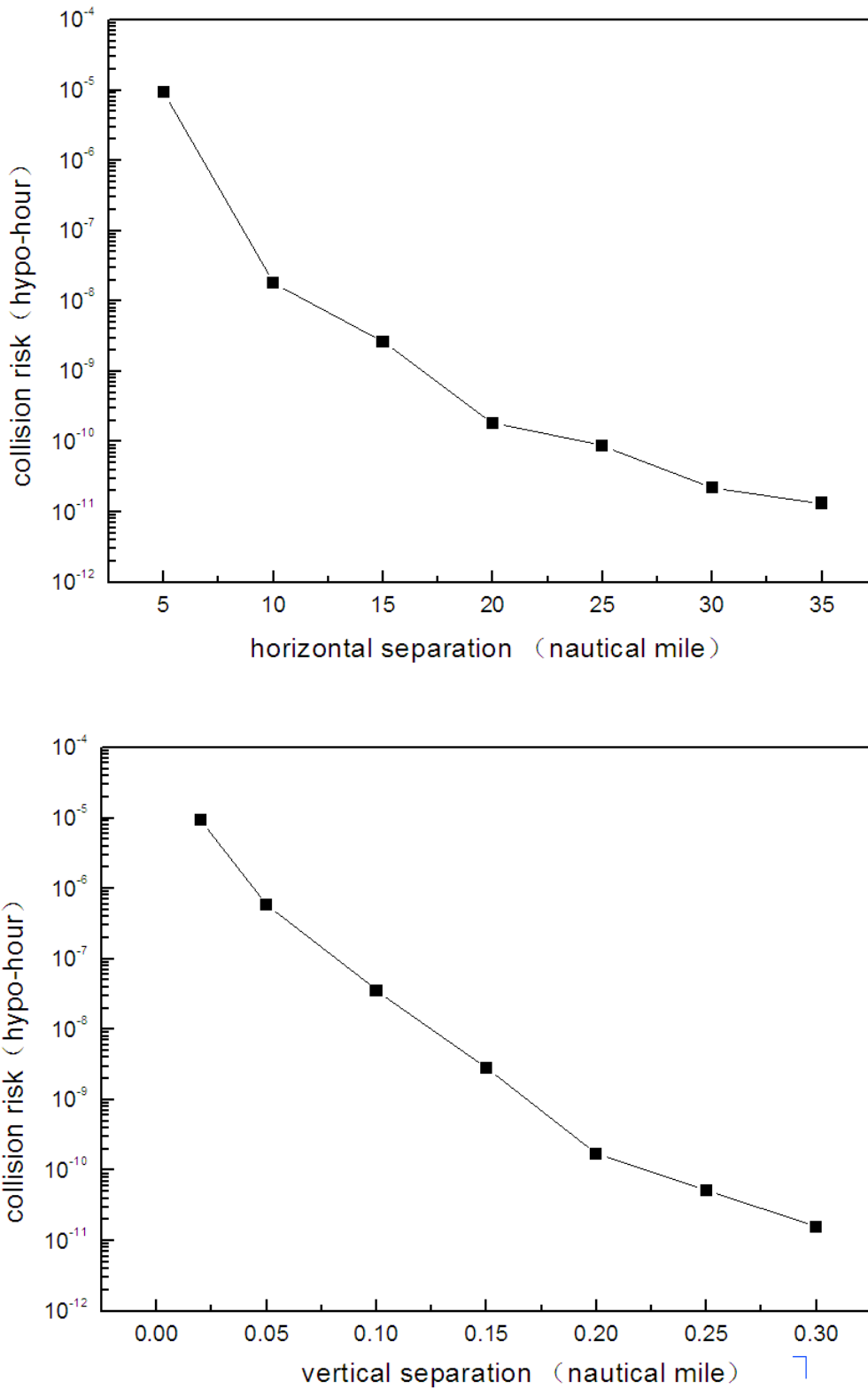


Figure 5 diagram of relationship between initial separation and collision risk

### V. Conclusion

The different collision risk models are established innovatively on the bases of different schemes to relief

from conflict, adjust course and velocity, according to the collision avoidance mechanism. The model considers influence of the positioning error to collision risk. And the numerical results show that the model is feasible. At the same time, the results also show that the course adjustment method for collision avoidance is the most reasonable, which fits to the actual situation. At last, the relationship between the separation and collision risk is analyzed and the different collision risk is provided on the different separation. So, the pilot can choose appropriate way to ensure flight safety according to the actual situation of aircraft, which makes the flight more flexible.

## **VI. Acknowledgements**

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## **References**

- [1] Final Report of RTCA Task Force 3-Free Flight Implementation[R].Washington DC:RTCA,1995
- [2] Reich.P.G. Analysis of long-range air traffic systems: separation standards III [J]. Journal of the Institute of Navigation. 19(3):331-347
- [3] Peter Brooker, Aircraft Collision Risk in the North Atlantic Region[J],Journal of the Operational Research Society,1984,Vol.35(8),695-703
- [4] Cox M.E., Harrison D., Moek G., ten Have J.M, et al. European Studies to Investigate the Feasibility of Using 1000 ft Vertical Separation Minima above FL 290, parts I II III [J], Journal of the Institute of Navigation. 1991, 44 (2): 171-183, 1992, 45 (1): 91-106, 1993, 46 (2): 245-261
- [5] ZHANG Zhao-ning,LIU Ji-min, Wang Li-li. Assessment of Longitudinal Collision Risk on Parallel Routes Based on Communication, Navigation and Surveillance Performances [J]. Journal of Southwest Jiaotong University, ,2009,44(6):918-925.
- [6] CAI Ming, ZHANG Zhao-ning, WANG Li-li. Research on Collision Risk in Free Flight[J]. Aeronautical Computing Technique, 2011,41(1):51-56.
- [7] LU Ting-ting, ZHANG Zhao-ning, LIU Ji-min. Research on safety assessment of collision risk in free flight[J]. Aeronautical Computing Technique, 2010,40(6):25-29.
- [8] CAI Ming, ZHANG Zhao-ning. Research on Exploring the most close time in Air Collision Avoidance System[A]. In 2010, transportation, colleges and universities graduate student academic BBS and proceedings of the science and technology innovation, 386-390.
- [9] ZHANG Zhao-ning, ZHOU Peng,LIU Jian-bin. Risk Model based on stochastic differential equation in Free Flight[J], Civil Aviation University of China Journal, 2012,30 (3): 1-5.
- [10] Zhaoning Zhang, Chang Sun, Peng Zhou. Safety Separation Assessment in Free Flight Based on Conflict Area[J]. Journal of Computers, 2012,7(10)2488-2495.
- [11] ZHANG Zhao-ning, WANG Li-li,LI Dong-bin. Flight Separation Assessment[M].Beijing: Science press, 2009, 126-163.